CONSTRAINED CONTROL OF A LOW POWER INDUSTRIAL GAS TURBINE

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Abstract: A constrained linear optimal control for a low-power industrial gas turbine based on input-output linearization is proposed in this paper. It uses a nonlinear state space model of the gas turbine in input-affine form based on first engineering principles. According to the control aims the nonlinear model is input-output linearized and an LQ servo controller is developed for the I/O linearized model. The hard constraints for the state and input variables are kept by applying constrained finite time linear optimal control.

Keywords: gas turbines, nonlinear systems, input-output linearization, constrained linear control.

1. INTRODUCTION

Gas turbines are important and widely used prime movers in transportation systems. Besides this main application area, gas turbines are found in power systems where they are the main power generators [1]. Therefore the modelling and control of gas turbines is of great practical importance.

Control techniques applied for gas turbines are most often based on linear controllers. These controllers are mainly variants of linear quadratic (LQ) controllers, e.g. in [2,3]. An LQ servo controller is applied to track a reference signal in [4]. LQG/LTR technique [5] and robust control system design has also been performed [6] for gas turbines.

In nonlinear control, however, the streamline is the application of adaptive (e.g. in [7]) and adaptive predictive (e.g. in [8]) control approaches, but there is a big lack of "classical" state-space nonlinear control. As a rare exception, in [9] the equation of mass flowrate of fuel (as the control input of a simplified single input-single output model) is determined by a nonlinear method.

In order to apply nonlinear state-space model based control, one has to develop a relative simple yet powerful dynamic model that is able to describe the nonlinear dynamic behavior of the gas turbine. A strongly nonlinear state space description of a low power gas turbine has been developed based on first engineering principles in an earlier paper [13]. Because of nonlinearities, however, the nonlinear dynamic analysis (controllability, observability and stability analysis) of the developed model can only be performed with difficulty, or in some cases it can not be computed symbolically at all [14].

An advanced and nonlinear control Lyapunov-function based block-structured controller [15] has been proposed for the same gas pilot-plant turbine that is used as a case study in this paper. In a PhD thesis [16], this nonlinear controller is compared with an LQ servo controller, as a reference case known from the literature. As a result of the comparison it is pointed out that the system controlled by the nonlinear control Lyapunov-function based controller exhibits similar or better qualitative and quantitative behavior, than the system controlled by the LQ-servo controller. However, the design of the nonlinear control Lyapunov-function based controller included some key heuristically performed steps that were strongly specific to the pilot-plant gas turbine model. Therefore, the need to apply an alternative technique a nonlinear controller based on input-output linearization has also been identified [17].

2. THE DYNAMIC MODEL OF THE GAS TURBINE

The main parts of a gas turbine include the inlet duct, the compressor, the combustion chamber, the turbine and the nozzle or the gas-deflector. The interactions between these components are fixed by the physical structure of the engine. The operation of all types of gas turbines is basically the same. The air is drawn into the engine through the inlet duct by the compressor, which compresses it and then delivers it to the combustion chamber. Within the combustion chamber the air is mixed with fuel and the mixture is ignited, producing a rise in temperature and hence an expansion of the gases. These gases are exhausted through the engine nozzle or the engine gas-deflector, but first pass through the turbine, which is designed to extract sufficient energy from them to keep the compressor rotating, so that the engine is self sustaining.

2.1 Modeling assumptions

In order to get a low order dynamic model suitable for control purposes the following modelling assumptions should be made.

General assumptions

- Constant physico-chemical properties are assumed in each main part of the gas turbine, such as specific heat at constant pressure and at constant volume, specific gas constant and adiabatic exponent.
- Heat loss (heat transmission, heat conduction, heat radiation) is neglected.

Other assumptions

- In the inlet duct a constant pressure loss coefficient (σ_I) is assumed.
- In the inlet and in the outlet of the compressor the mass flow rates are the same: $\nu_{Cin}=\nu_{Cout}=\nu_C$, and there is no energy storage effect: $U_2=constant$.
- In the combustion chamber constant pressure loss coefficient (σ_{Comb}) and constant efficiency of combustion (η_{Comb}) are assumed; the enthalpy of fuel is neglected, and the combustion chamber is assumed to be a perfectly stirred region (balance volume). It means that a finite dimensional concentrated parameter model is developed and the value of the variables within this balance volume is equal to that at its outlet.
- In the inlet and in the outlet of the turbine the mass flow rates are the same: $\nu_{Tin}=\nu_{Tout}=\nu_T$, and there is no energy storage effect: $U_4=constant$.
- In the gas-deflector a constant pressure loss coefficient (σ_N) is assumed.

2.2 Conservation balances

The nonlinear state equations are derived from the laws of conservation principles. Dynamic equations come from the conservation balances constructed for the overall mass m and internal energy. The development of the model equations is performed in the following steps.

Conservation balance of the total mass:

$$\frac{dm}{dt} = \nu_{in} - \nu_{out}$$

Conservation balance of the internal energy, where the heat energy flows and the power terms are also taken into account:

$$\frac{dU}{dt}=v_{in}i_{in}-v_{out}i_{out}+Q+P$$

We can transform the above energy conservation equation to its intensive variable form by considering the dependence of the internal energy on the measurable temperature:

$$\frac{dU}{dt}=c_v\frac{d}{dt}(Tm)=c_vT\frac{dm}{dt}+c_vm\frac{dT}{dt}$$

From the two equations above we get a state equation for the temperature as state variable:

$$\frac{dT}{dt}=\frac{v_{in}i_{in}-v_{out}i_{out}+Q+P-c_vT\ \left(v_{in}-v_{out}\right)}{c_vm}$$

The ideal gas equation *(pV=mRT)* is used together with the two balance equations above to develop an alternative state equation for the pressure:

$$\frac{dp}{dt}=\frac{R}{c_vV}\left(v_{in}i_{in}-v_{out}i_{out}+Q+P\right)$$

Conservation balance of the mechanical energy of the compressor-turbine shaft:

$$\frac{dE_{shaft}}{dt}=v_Tc_{pgas}\left(T_3-T_4\right)\eta_{mech}-v_Cc_{pair}\left(T_2-T_1\right)-2\pi\frac{3}{50}M_{load}$$

2.3 Simplified nonlinear model in input-affine form

From the above equations (using the constitutive algebraic equations, too), the nonlinear input-affine form of the gas turbine model has the following structure.

$$\dot{x}=f(x)+g(x)u$$

$$y=h(x)$$

where the state vector is $x=\begin{bmatrix}m_{Co} & p_3 & n\end{bmatrix}^T$, the input variable is $u=v_{fuel}$. The set of possible disturbances is $d=\begin{bmatrix}p_1 & T_1 & M_{load}\end{bmatrix}^T$, and the measurable output vector is $y=\begin{bmatrix}T_4 & p_3 & n\end{bmatrix}$. The nonlinear functions in the input-affine model have the form

$$f(x)=\begin{bmatrix}f_1(x_1,x_2,x_3,d_1,d_2)\\ f_2(x_1,x_2,x_3,d_1,d_2)\\ f_3(x_1,x_2,x_3,d_1,d_2,d_3)\end{bmatrix}$$

$$g(x)=\begin{bmatrix}const_1\\ const_2\\ 0\end{bmatrix}$$

$$h(x)=\begin{bmatrix}h_1(x_1,x_2,x_3,d_1)\\ x_2\\ x_3\end{bmatrix}.$$

The nonlinear model of the turbine is valid within the following operating domain:

$$0.00305\,kg\leqslant x_1\leqslant 0.00835\,kg$$

$$154837\,Pa\leqslant x_2\leqslant 325637\,Pa$$

$$650\,1/s\leqslant x_3\leqslant 866\,1/s$$

The explanation of the variables and parameters of the simplified model is shown in Table 1.

Not.	Variable name / Units
m_{Comb}	mass in combustion chamber *[kg]*
P_3	turbine total inlet pressure *[Pa]*
N	rotational speed *[1/s]*
v_{fuel}	mass flowrate of fuel *[kg/s]*
P_1	compressor inlet total pressure

	[Pa]
T_1	compressor inlet total temperature *[K]*
M_{load}	loading moment *[Nm]*
T_4	turbine outlet total temperature *[K]*

Table 1. Most important variables and constants of the model

3. NONLINEAR CONTROLLER DESIGN

3.1 Input/output linearization

The (input-output) linearized model enables that the turbine is asymptotically stabilized with a simple linear quadratic (LQ) controller.

It is easy to compute that the relative degree of the turbine model with output being the (centered) rotational speed is *r=2* in a neighborhood of the operating point, therefore the (nonlinear) zero dynamics is one dimensional. Consider the following nonlinear feedback which contains a zeroing input with an additional external input function v:

$$u=\alpha(x)+\beta(x)v=\frac{-L_f^{2h}(x)}{L_g L_f h(x)}+\frac{1}{L_g L_f h(x)}v$$

Applying this input function to the equations of the simplified model (Eqs. (23-25)) we obtain that d^2x_3/dt^2=v, therefore the input-output linearized system can be represented by the following transfer function from *v* to *y*:

$$H(s)=\frac{1}{s^2} \quad .$$

3.2 LQ-servo controller

Our aim here is to design a controller that is able to track a constant reference. For this purpose, we extend our linearized model with the following differential equation:

$$\dot{e}=v_{ref}-y=v_{ref}-x_3$$

For the asymptotic stabilization of the augmented system we design a standard linear quadratic (LQ) controller. The weighting matrices of the controller were chosen as

$$Q=\begin{bmatrix}1 & 0 & 0\\ 0 & 10000 & 0\\ 0 & 0 & 20000\end{bmatrix}, R=1$$

The resulting stabilizing feedback gain was $k=[18.004 \quad 161.56 \quad -447.21]$.

3.3 Constrained linear optimal control

The constrained linear optimal control technique uses the following discrete time LTI system form:

$$x(k+1)=Ax(k)+Bu(k), \quad y(k)=Cx(k)+Du(k)$$

The so-called Constrained Finite Time Optimal Control Problem (see, e.g. [10-11]) is to find an input sequence $\{u(0),\dots u(N-1)\}$ such that it minimizes the cost function

$$J(x,u)=x(N)^T P_N x_N+\sum_{k=1}^{N-1} u(k)^T R u(k)+x(k)^T Q x(k)$$

subject to the constraints

$$u_{min}\leqslant u(k)\leqslant u_{max}$$
$$y_{min}\leqslant y(k)\leqslant y_{max}$$
$$Hx(k)\leqslant K$$

where P_N, Q and R are positive definite symmetric weighting matrices, *H* and *K* are matrices defining the polytopic region of the state space inside which the state vector has to evolve. To solve the constrained control problem for the LQ-servo controlled gas-turbine model, the Multi-Parametric Toolbox (MPT) of Matlab [12] has been used.

The constraints for the rotational speed, its derivative and the input were the following:

$$800 \leqslant n \leqslant 866 \quad 1/s$$

$$-500 \leqslant \dot{n} \leqslant 500 \quad 1/s^2$$

$$0.00584 \leqslant v_{fuel} \leqslant 0.02419 \quad kg/s$$

4. SIMULATION RESULTS

Fig. 1 shows the state variables of the system as the loading moment decreases from 50 Nm to 30 Nm at t=1s and then from 30 Nm to 20 Nm at t=3s.

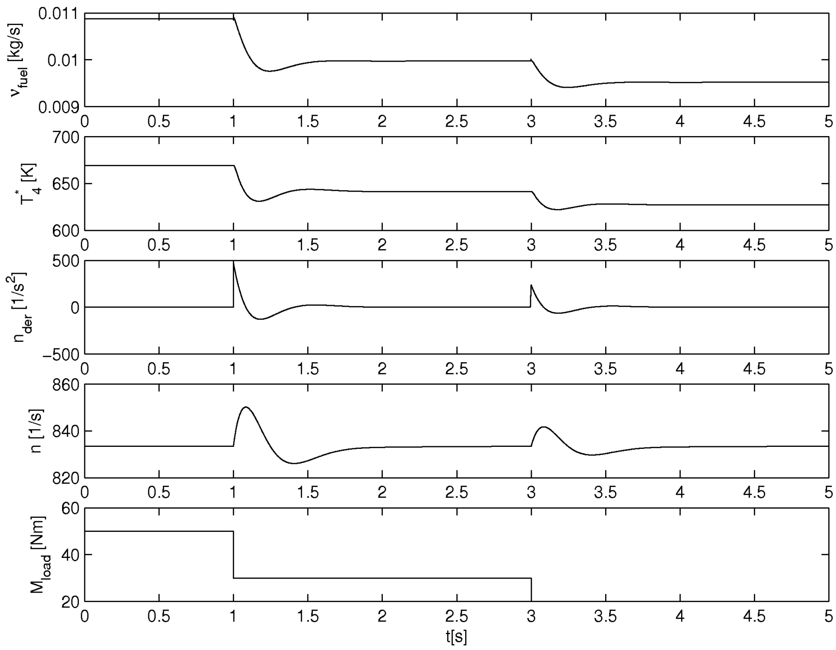

Figure 1. Response of the controlled system to step-like loading moment changes

It is visible that the control input, the rotational speed and its time-derivative remain within the predefined range.

5. CONCLUSIONS

A constrained linear optimal control has been proposed in this paper for a low-power industrial gas turbine based on input-output linearization. The nonlinear state space model of the gas turbine in input-affine form is based on first engineering principles. In the first step of controller design this nonlinear model is input-output linearized and an LQ servo controller is developed for the linearized model. To satisfy the hard constraints on the state and input variables, the Finite Time Constrained Optimal Control Problem is solved for the system. The simulation results show that good time-domain performance can be reached together with keeping the given input and state constraints.

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